L-functions and Arithmetic
Conference Program

Monday, June 13

9:15-9:30am Barry Mazur, Harvard University
Welcoming remarks

9:30-10:30am John Coates, University of Cambridge
Title: "On the conjecture of Birch and Swinnerton-Dyer for elliptic curves with complex multiplication"

Abstract: The lecture will describe some recent joint work with Y. Kezuka, Y. Li and Y. Tian. Let E be any elliptic curve defined over $\mathbb{Q}$ which is a quadratic twist of the modular curve $X_0(49)$. Let $E(\mathbb{Q})$ be the group of rational points of E, Ш$(E/\mathbb{Q})$ its Tate-Shafarevich group, and $L(E/\mathbb{Q}, s)$ its complex L-series. I will begin by discussing the proof of the following generalization of earlier work of K. Rubin and C. D. Gonzalez-Aviles.

Theorem. *We have* $L(E/\mathbb{Q}, 1) \neq 0$ *if and only if both* $E(\mathbb{Q})$ *and the* 2*-primary subgroup of* Ш$(E/\mathbb{Q})$ *are finite. When these equivalent conditions hold, the full Birch-Swinnerton-Dyer conjecture is valid for* E.

Then I will discuss a partial generalization to a large family of quadratic twists of the Gross curves $A(q)$, with complex multiplication by the ring of integers of an imaginary quadratic field $K = \mathbb{Q}(\sqrt{-q})$, where q is any prime which is congruent to 7 modulo 8.

10:30-11:00am Coffee Break

11:00-12:00pm Bjorn Poonen, MIT
Title: "Selmer group heuristics"

Abstract: I will give a survey of some of the statements that have been proved or conjectured about the distribution of Selmer groups of elliptic curves over global fields.

12:00-2:00pm Lunch

2:00-3:00pm Chris Skinner, Princeton University
Title: "Special values of *L*-functions of elliptic curves and modular forms"

Abstract: The talk will describe some recent work towards proving the p-part of the conjectured Birch-Swinnerton-Dyer formula for the central value of the L-function of an elliptic curve in cases of rank 0 or 1, along with generalizations to modular forms.

3:00-3:30pm Coffee Break

3:30-4:30pm Ralph Greenberg, University of Washington

Title: "Minimal Behavior of Mordell-Weil groups and Selmer groups in towers of number fields"

Abstract: If E is an elliptic curve over $\mathbb{Q}$, K is finite Galois extension of $\mathbb{Q}$, χ is an irreducible real-valued character of Gal($K/\mathbb{Q}$), and the sign in the functional equation for the L-function $L(s, E, \chi)$ is -1, then $L(1, E, \chi)$ should vanish and χ should conjecturally occur as a constituent in $E(K) \otimes \mathbb{Q}$. If K runs over a tower of number fields, then one can ask whether the growth of the Mordell-Weil groups or the Selmer groups is entirely explained by the signs in the corresponding functional equations. This type of behavior is sometimes possible and sometimes not possible. We will discuss this question in various situations in this talk.

Tuesday, June 14

9:00-10:00am Joël Bellaïche, Brandeis University

Title: "Image of big Galois representations and modular forms mod *p*"

Abstract: In recent years, the structure of the Hecke algebras of modular forms (of fixed level and any weight) modulo an odd prime p has been determined. Those algebras carry a natural representation or pseudo-representation, whose image can be precisely described. I will explain that description and show how it can be used to prove uniform bounds on the density of non-vanishing coefficients of modular forms mod p.

10:00-10:30am Coffee Break

10:30-11:30am Melanie Matchett Wood, University of Wisconsin

Title: "Nonabelian Cohen-Lenstra Heuristics and Function Field Theorems"

Abstract: The Cohen-Lenstra Heuristics conjecturally give the distribution of class groups of imaginary quadratic fields. Since, by class field theory, the class group is the Galois group of the maximal unramified abelian extension, we can consider the Galois group of the maximal unramified extension as a non-abelian generalization of the class group. We will explain non-abelian analogs of the Cohen-Lenstra heuristics due to Boston, Bush, and Hajir and work, some joint with Boston, proving cases of the non-abelian conjectures in the function field analog.

11:30am-12:00pm Coffee Break

12:00pm-1:00pm Mirela Çiperiani, University of Texas at Austin

Title: "Local points of supersingular elliptic curves on $\mathbf{Z}_p$-extensions"

Abstract: Work of Kobayashi and Iovita-Pollack describe how local points of supersingular elliptic curves on ramified $\mathbf{Z}_p$-extensions of $\mathbf{Q}_p$ split into two strands of even and odd points.

We will discuss a generalization of this result to $\mathbf{Z}_p$-extensions that are localizations of anticyclotomic $\mathbf{Z}_p$-extensions over which the elliptic curve has non-trivial CM points.

1:00pm-2:30pm Lunch

2:30pm-3:30pm Jan Nekovář, Univ. Pierre-et-Marie Curie - Paris VI

Title: "Some remarks on the Conjecture of Birch and Swinnerton-Dyer"

Abstract: We are going to discuss several topics related to the conjecture of Birch and Swinnerton-Dyer.

3:30pm-4:00pm Coffee Break

4:00-5:00pm Kiran Kedlaya, UCSD

Title: "Some questions about the fields of definition of endomorphisms of abelian varieties"

Abstract: Let A be an abelian variety of genus g over a number field K. Let L be the minimal extension of K over which all of the geometric endomorphisms of A become defined. It is an old observation of Silverberg that L/K is a finite Galois extension whose order can be bounded in terms of the orders of certain matrix groups over finite fields (by analogy with the Schur-Minkowski theorem on finite groups of integer matrices). We discuss an attempt to sharpen this bound, with somewhat surprising results.

Wednesday, June 15

9:30-10:30am Andrew Wiles, University of Oxford

Title: "Properties of compatible systems"

Abstract: I will talk about properties of compatible systems of Galois representations.

10:30-11:00am Coffee Break

11:00-12:00pm Massimo Bertolini, Universität Duisburg-Essen

Title: "*p*-adic *L*-functions and rational points on elliptic curves I. Beilinson and Beilinson-Flach elements in *p*-adic families"

Abstract: Joint abstract with Henri Darmon. The main title of these lectures refers to the two almost eponymous papers written by Karl Rubin (Inventiones, 1991) and by Bernadette Perrin-Riou (Annales de l'Institut Fourier, 1993). Rubin's remarkable result relates the value of the Katz p-adic L-function of an imaginary quadratic field at a point lying outside its range of classical interpolation to the p-adic logarithm of a global point on the idoneous elliptic curve with complex multiplication. Its proof rests on a careful comparison between the Euler systems of elliptic units and Heegner points. Shortly afterwards, Bernadette Perrin-Riou proposed a conjectural extension of Rubin's formula to arbitrary (modular) elliptic curves, inspired by Kazuya Kato's approach to the Birch and Swinnerton-Dyer conjecture based on his Euler system of Beilinson elements in p-adic families. This series of two lectures proposes a framework for Rubin's formula and Perrin-Riou's conjecture, and proves the latter in the setting of elliptic curves over $\mathbf{Q}$ twisted by quadratic Dirichlet characters. The proof rests on a comparison between

I. Kato's global cohomology classes and the "generalised Kato classes" introduced in the earlier work of Victor Rotger and the two speakers on Beilinson-Flach elements in p-adic families;

II. These generalized Kato classes and Heegner points, based on a special case of the "elliptic Stark conjecture" proved in collaboration with Alan Lauder and Victor Rotger, whose proof builds on a p-adic Gross-Zagier formula obtained in an earlier collaboration with Kartik Prasanna.

These comparisons will be described in the first and second lectures respectively.

12:00-2:00pm Lunch

2:00-3:00pm David Burns, King's College London

Title: "On the theory of higher rank Euler and Kolyvagin systems"

Abstract: I will give an overview of some recent results concerning the theory of higher rank Kolyvagin systems of Mazur and Rubin. In particular, I will discuss how a natural higher rank Kolyvagin derivative operator clarifies the (hitherto 'mysterious') connection to the associated theory of higher rank Euler systems and also briefly explore applications of the general theory. This is joint work with Takamichi Sano.

3:00-3:30pm Coffee Break

3:30-4:30pm Haruzo Hida, UCLA

Title: "Anti-cyclotomic Cyclicity and Gorenstein-ness of Hecke algebras"

Abstract: Under mild conditions, we prove certain Gorenstein property of Hecke algebras is closely related to cyclicity of the Iwasawa module for the anticyclotomic $\mathbf{Z}_p$-extension over an imaginary quadratic field. If such ring theoretic result holds, the characteristic ideal of the Iwasawa module determines the isomorphism class of the Iwasawa module, as expected by Iwasawa.

Thursday, June 16

9:00-10:00am Henri Darmon, McGill University

Title: "p-adic L-functions and rational points on elliptic curves II. Beilinson-Flach elements and Heegner points"

Abstract: Joint abstract with Massimo Bertolini. The main title of these lectures refers to the two almost eponymous papers written by Karl Rubin (Inventiones, 1991) and by Bernadette

Perrin-Riou (Annales de l'Institut Fourier, 1993). Rubin's remarkable result relates the value of the Katz p-adic L-function of an imaginary quadratic field at a point lying outside its range of classical interpolation to the p-adic logarithm of a global point on the idoneous elliptic curve with complex multiplication. Its proof rests on a careful comparison between the Euler systems of elliptic units and Heegner points. Shortly afterwards, Bernadette Perrin-Riou proposed a conjectural extension of Rubin's formula to arbitrary (modular) elliptic curves, inspired by Kazuya Kato's approach to the Birch and Swinnerton-Dyer conjecture based on his Euler system of Beilinson elements in p-adic families. This series of two lectures proposes a framework for Rubin's formula and Perrin-Riou's conjecture, and proves the latter in the setting of elliptic curves over **Q** twisted by quadratic Dirichlet characters. The proof rests on a comparison between

I. Kato's global cohomology classes and the "generalised Kato classes" introduced in the earlier work of Victor Rotger and the two speakers on Beilinson-Flach elements in p-adic families;

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These comparisons will be described in the first and second lectures respectively.

10:00-10:30am Coffee Break

10:30-11:30am Sarah Zerbes, University College London

Title: "New Euler systems for automorphic Galois representations"

Abstract: I will outline the construction of new Euler systems for certain automorphic Galois representations. I will describe the following cases: the Asai Galois representation for a quadratic Hilbert modular form, the spin representation for a genus 2 Siegel modular form, and the standard representation of an automorphic eigenform for U(2,1). This is joint work with Antonio Lei, David Loeffler and Chris Skinner.

11:30am-12:00pm Coffee Break

12:00pm-1:00pm Manjul Bhargava, Princeton University

Title: "A positive proportion of plane cubics fail (resp. satisfy) the Hasse principle"

Abstract: In 1951, Selmer showed that the plane cubic curve cut out by $3x^3+4y^3+5z^3$ has no rational point, despite having a point locally at all places of **Q**. That is, this curve fails the Hasse principle.

How rare are such counterexamples to the Hasse principle among plane cubic curves? In this talk, using recent work with Arul Shankar and Christopher Skinner, we show that in fact a positive proportion of all plane cubics (when ordered by the heights of their defining equations) fail the Hasse principle; moreover, a positive proportion of plane cubics satisfy the Hasse principle and possess a rational point.