

$$\begin{array}{ccccccc}
& & \pi^* V_d^g & & V_d^g & & \rho^* V_d^g & & V \\
& & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
W_d & \xleftarrow{\phi} & M_d^g(X) & \xrightarrow{\pi} & \mathcal{M}_{g,0}(d, X) & \xleftarrow{\rho} & \mathcal{M}_{g,1}(d, X) & \xrightarrow{ev} & X
\end{array}$$

W_d = linearized moduli,

$M_d^g(X)$ = nonlinear moduli,

$\mathcal{M}_{g,0}(d, X)$ = moduli space of stable maps

fiber of V_d^g = $H^0(C, f^*V) \oplus H^1(C, f^*V)$

ω = $\pi^* b(V_d^g)$,

where b is a multiplicative characteristic class

$$Q_d = \phi_!(\omega)$$