An International Conference to
Celebrate the Birthday of Shing-Tung Yau
August 27-September 1, 2008

Spaces of Measures and Geometry

Jean-Pierre Bourguignon
(CNRS-IHÉS)

Abstract: Motivated by geometric (and also physical) problems, Misha Gromov advocated the loosening of the relation between a metric and a measure that is familiar in Riemannian geometry where the metric is obtained by integrating the length of shortest paths and the volume element is induced by the metric. This lead to the study of Metric Measured Spaces.

Motivated by optimal transport, Felix Otto and Cédric Villani, among others, made evident the importance of the geometry of spaces of measures. Their efforts met earlier ones made by probabilists such as Dominique Bakry and Michel Émery, who looked for conditions ensuring the validity of log-Sobolev inequalities. Maxim Kontsevitch and John Lott rediscovered some of these facts in their attempts to generalize the notion of Ricci curvature to non smooth metrics.

This very rich circle of ideas has become a very active area of research allowing to revisit some classical domains of Analysis, such as solving Monge Ampère equations and making evident the importance of entropy functionals in this context. The intricate mixing of ideas, coming from diverse fields, is quite remarkable in that it brings together fundamental as well as applied viewpoints on questions that initially were considered foreign to one another.

The purpose of the lecture is to present this process as well as some recent results in the area.