The theory of H-laminations and CMC foliations

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Abstract: I will discuss some recent developments in the theory of embedded minimal and constant mean curvature hypersurfaces. One of these results is the is the Stable Limit Leaf Theorem whose proof I will give in the case of minimal laminations of 3-manifolds. I will cover several interesting applications of this result to the theory of H-laminations and CMC foliations. Some interesting consequences include the proof that a CMC foliation F of a homogeneously regular 3-manifold N has a bound on the norm of the second fundamental form of its leaves that only depends on a bound of the absolute sectional curvature of N. This curvature estimate leads to sharp absolute mean curvature bounds of the leaves of the possible foliations. For example, in hyperbolic three space I will prove that this bound is 1. I will also classify all weak CMC foliations of R^3 with at most a closed countable set S of singularities by showing that all leaves are spheres and planes; in particular $|S| < 3$. For a homogeneously regular 5-manifold N of absolute sectional curvature at most 1, these results generalize to show that every codimension one CMC foliation of N has leaves with absolute mean curvature less than 2; this last mean curvature estimate depends in part on a classical result of Schoen-Simon-Yau on the classical Bernstein problem.