JDG 2017 schedule
Friday April 28, 2017

2:00–2:30 Opening Comments (30 minutes)

2:30–3:20 pm
Cliff Taubes (Harvard University)
The behavior of sequence of solutions to the Vafa-Witten equations
The Vafa-Witten equations on an oriented Riemannian 4-manifold are first order, non-linear equations for a pair of connection on a principle SO(3) bundle over a 4-manifold and a self-dual 2-form with values in the associated Lie algebra bundle. This talk will describe a theorem about the behavior of sequences of solutions to the Vafa-Witten equations which have no convergent sub-sequence. This theorem says in part that a renormalization of a subsequence of the self-dual 2-form components of any given solution sequence converges on the complement of a closed set with Hausdorff dimension at most 2; and the limit defines a harmonic 2-form with values in a real line bundle. This behavior generalizes Karen Uhlenbeck's compactness theorem for the self-dual Yang-Mills equations; it is similar to what happens in other first order generalizations of the Seiberg-Witten/self-duality equations.

3:35–4:25 pm
Chiu Chiu Melissa Liu (Columbia University)
Mirror Symmetry and Topological Recursion
The Remodeling Conjecture proposed by Bouchard-Klemm-Mariño-Pasquetti provides a precise correspondence between open-closed Gromov-Witten invariants of a symplectic toric Calabi-Yau threefold and the invariants of the mirror curve defined by Eynard-Orantin topological recursion. It can be viewed as a version of all genus open-closed mirror symmetry. I will present a proof of the conjecture and describe its implications on the structure of higher genus Gromov-Witten invariants, based on joint work with Bohan Fang and Zhengyu Zong.

4:40–5:30 pm
Cumrun Vafa (Harvard University)
String Theory and Homological Invariants for 3-Manifolds
In this talk I review the recent progress made in defining homological invariants for 3-manifold using string theory constructions. This generalizes the constructions of homological invariants for knots using M5 branes, to the case of 3-manifolds.

5:45–6:35 pm
Dan Freed (University of Texas)
Complex Chern-Simons invariants of 3-manifolds and abelianization
A hyperbolic 3-manifold M carries a flat PSL(2;C)-connection whose Chern-Simons invariant has been much studied since the early 1980's. For example, its real part is the volume of M. Explicit formulas in terms of a triangulation involve the dilogarithm. In joint work with Andy Neitzke we use 3-dimensional spectral networks to abelianize the computation of complex Chern-Simons invariants. The locality of the classical Chern-Simons invariant, expressed in the language of topological field theory, plays an important role.

Saturday April 29, 2017

9:00–9:50 am
Frances Kirwan (Oxford University)
Variation of non-reductive geometric invariant theory
Mumford's geometric invariant theory (GIT) provides a method for constructing quotient varieties for linear actions of reductive groups on projective varieties. The GIT quotient depends on the choice of linearisation for the group action, and this dependence was described using 'variation of GIT' (VGIT) by Thaddeus and Dolgachev & Hu in the 1990s. GIT has been extended to non-reductive actions; many of the nice features of classical GIT fail in general, but are satisfied given the extra data of a graded linearisation for an action of a linear algebraic group with graded unipotent radical. The aim of this talk is to describe this picture and a version of VGIT which applies to it (joint work with Gergely Berczi and Joshua Jackson).

10:05–10:55 am
Mina Aganagic (University of California, Berkeley)
Mathematical applications of little string theory
I will describe applications of a six dimensional string theory to the Geometric Langlands Program and to the Knot Categorification Program. This is based on joint works with Edward Frenkel and Andrei Okounkov.

11:10–12:00 pm
Caucher Birkar (University of Cambridge)
Singularities and Fano varieties in birational geometry
Fano varieties constitute a fascinating class of algebraic varieties that are important in birational geometry and beyond. On the other hand, studying mild singularities is an indispensable feature of modern birational geometry. In this talk I will try to explain how one interwines the two subjects to prove various local and global boundedness statements regarding linear systems on varieties and families of Fano varieties.

12:00–1:30 LUNCH BREAK

1:30–2:20 pm
Duong Phong (Columbia University)
Supersymmetric string vacua with torsion and geometric flows
In 1986, a system of equations for compactifications of the heterotic string which preserve supersymmetry was proposed independently by C. Hull and A. Strominger. They are more complicated than the Calabi-Yau compactifications proposed earlier by P. Candelas, G. Horowitz, A. Strominger, and E. Witten, because they allow non-vanishing torsion and they incorporate terms which are quadratic in the curvature tensor. As such they are also particularly interesting from the point of view of both non-Kaehler geometry and the theory of non-linear partial differential equations. While the complete solution of such partial differential equations seems out of reach at the present time, we describe progress in developing a new general approach based on geometric flows which shares some features with the Ricci flow. In particular, this approach can recover the well-known non-perturbative solutions found in 2006 by J.X. Fu and S.T. Yau. This is joint work with S. Picard and X.W. Zhang.

2:35–3:25 pm
Richard Schoen (University of California, Irvine)
The Einstein constraint equations
Solutions of the Einstein equations evolve on the constraint manifold which is an infinite dimensional subset of tensors on a three manifold M which satisfy an under-determined geometric system of equations. There are many approaches to solving these equations and attempts at understanding the structure of the manifold. In this talk we will survey this topic including recent work on constructing localized solutions and special types of black hole solutions.

3:45–4:35 pm
Fernando Coda Marques (Princeton)
The space of cycles, a Weyl's law for minimal hypersurfaces and Morse index estimates
The space of cycles in a compact Riemannian manifold has very rich topological structure. The space of hypercycles, for instance, taken with coefficients modulo two, is weakly homotopically equivalent to the infinite dimensional real projective space. This reveals the existence of nontrivial k-parameter sweepouts for every k. We will discuss a proof of a Weyl's law conjectured by Gromov (joint work with Liokumovich and Neves) in which the eigenvalues of the Laplacian are replaced by the areas of minimal hypersurfaces constructed by minimax methods. We will also discuss current work with Neves about Morse index bounds in the min-max theory of minimal surfaces and the problem of multiplicity.

4:50–5:40 pm
Ciprian Manolescu (University of California, Los Angeles)
Homology cobordism and triangulations
Abstract: In the 1970's, Galewski-Stern and Matumoto studied the existence and the classification of triangulations on topological manifolds of dimension at least five. They reduced these problems to questions about the three-dimensional homology cobordism group, \Theta^H_3, and the Rokhlin homomorphism from this group to Z/2. The structure of the homology cobordism group is still unknown, but some information can be obtained using tools from gauge theory and symplectic geometry, such as the Seiberg-Witten Floer spectrum and involutive Heegaard Floer homology. I will describe the proof of the existence of non-triangulable high-dimensional manifolds (using gauge theory), and some open problems.

7:00 pm Banquet at Loeb House
By invitation only

Sunday April 30, 2017

9:00–9:50
Jun Li (Stanford University)
New recursion relation for GW of quintic CY via Mixed-Spin-P fields
We introduce the notion of Mixed-Spin-P fields as an algebro-geometric model interpolating all genus GW and FJRW invariants of the quintic CY threefolds, realizing the vision of Witten's transformation relating these two theories. This theory of mixed-P-fields produces recursive relations among GW and FJRW invariants. These relations hopefully will provide the mean to solve all genus GW invariants of quintic CY threefolds. This is a joint work with HL. Chang, WP. Li and CC. Liu.

10:05–10:55
Si Li (Tsinghua University)
Vertex algebras, quantum master equation and mirror symmetry
We develop the effective Batalin-Vilkovisky quantization theory for chiral deformation of two dimensional conformal field theories. We establish an exact correspondence between renormalized quantum master equations for effective functionals and Maurer-Cartan equations for chiral vertex operators. As an application, we explain a universal approach to KdV type integrable hierarchies via B-twisted topological string field theory. This leads to an exact solution of quantum B-model (BCOV theory) in complex one dimension that solves the higher genus mirror symmetry conjecture on elliptic curves.

11:10–12:00
Yujiro Kawamata (University of Tokyo)
Birational geometry and derived categories
I will talk about the recent progress on the DK conjecture connecting birational geometry and the derived categories, and related conjectures such as DL conjecture, etc. I will also discuss two kinds of factorizations of birational maps; those into flips, flops and divisorial contractions according to the minimal model program, and more traditional factorizations into blow-ups and blow-downs with smooth centers.

12:00–1:30 LUNCH BREAK

1:30–2:20pm
Alena Pirutka (Courant Institute, New York University)
Irrationality problems
Let X be a projective algebraic variety, the set of solutions of a system of homogeneous polynomial equations. Several classical notions describe how ``unconstrained'' the solutions are, i.e., how close X is to projective space: there are notions of rational, unirational and stably rational varieties. Over the field of complex numbers, these notions coincide in dimensions one and two, but diverge in higher dimensions. In this talk I will discuss classical and recent advances in this area, examples and deformation properties

2:35–3:25pm
Mu-Tao Wang (Columbia University)
Linear stability of Schwarzschild black hole: the Cauchy problem of metric coefficients
The Schwarzschild solution of the vacuum Einstein equation in general relativity is the unique static solution that represents an isolated gravitating system of a single black hole. Studies, both theoretically

and experimentally, of such a system are modeled on the Schwarzschild solution and its perturbation. The stability of the Schwarzschild solution is thus of utmost importance. I will address the linear stability of the Schwarzschild solution, which has a long history and rich literature involving the works of both physicists and mathematicians, and culminating in the recent breakthrough of Dafermos-Holzegel-Rodnianski. In joint work with Pei-Ken Hung and Jordan Keller, we provide a different and simpler proof that reveals the underlying geometric structure of the vacuum Einstein equation at a more elementary level.

3:45–4:35pm
Jean-Pierre Demailly (Grenoble)
L² Extension theorem for sections defined on non reduced analytic subvarieties
The goal of the talk will be to discuss L² extension properties for holomorphic sections of vector bundles satisfying weak semi-positivity properties. Using techniques borrowed from recent proofs of the Ohsawa-Takegoshi extension theorem, we obtain several surjectivity results for the restriction morphism to a non necessarily reduced subvariety, provided the latter is defined as the zero variety of a multiplier ideal sheaf. These extension results are derived from L² approximation techniques, and they hold under rather general geometric assumptions.

4:50–5:40pm
Bong Lian (Brandeis University)
The Riemann-Hilbert problem for period integrals--recent applications
I will discuss two recent applications of the Riemann-Hilbert problem for periods of CY manifolds. One of them deals with the hyperplane conjecture for toric hypersurfaces that goes back to the mid 1990's. The second application provide descriptions for zeros of derivatives of generalized hypergeometric functions. The lecture is based on joint work with A. Huang, S.-T. Yau and M. Zhu.

5:55–6:45 pm
Steve Zelditch (Northwestern University)
Local and global analysis of nodal sets
Nodal sets are zero sets of eigenfunctions of the Laplacian on a Riemannian manifold. Local analysis studies nodal sets in small balls, ignoring the global geometry. Global analysis exploits the dynamics of the geodesic flow to obtain information on nodal sets. First, I will describe the recent proof by Alexander Logunov of Yau's lower bound conjecture for hypersurface volumes of nodal sets. It is a local proof based mainly on the combinatorics of the Donnelly-Fefferman doubling exponent bounds. Second, I will describe recent results on numbers of nodal domains on surfaces of non-positive curvature. These results are based on the ergodicity of the geodesic flow (joint work in part with Junehyuk Jung, related work by Ghosh-Reznikov-Sarnak).

Monday May 1, 2017

9:00–9:50 am
Richard Hamilton (Columbia University)
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10:05–10:55 am
Huai-Dong Cao (Lehigh University)
Geometry and Stability of Ricci Solitons
Ricci solitons, introduced by R. Hamilton in the mid-1980s, are self-similar solutions to the Ricci flow and often appear as singularity models of the Ricci flow. Ricci solitons are also natural extensions of Einstein metrics and are critical points of certain functionals defined by Perelman and others. In this talk I shall survey some recent developments on gradient shrinking Ricci solitons, including their geometry, classifications, and stability.

11:10–12:00 pm
Larry Guth (Massachusetts Institute of Technology)
Efficiently contracting contractible maps
Suppose that f is a contractible map from the unit m-sphere to the unit n-sphere with Lipschitz constant L. Is it possible to choose a null-homotopy with Lipschitz constant bounded by a reasonable function of L? Gromov posed this question about twenty years ago. For special choices of m and n, he constructed homotopies with Lipschitz constant at most C(m,n) L. But for most dimensions, the bounds that were known until recently were astronomical -- towers of exponentials in L. In the last year, Chambers, Dotterer, Manin, Ferry, and Weinberger constructed null-homotopies with nearly sharp Lipschitz constants. I will give a little background about the problem and then discuss their work.

12:00–1:30 pm LUNCH

1:30–2:20 pm
Simon Donaldson (Simons Center for Geometry and Physics, Stony Brook University)
Variational problems related to special holonomy
The starting part for the talk is work of Hitchin, giving a variational description of special geometric structures in dimensions 6,7,8 in terms of a volume functional. (See for example "The geometry of 3-forms in six dimensions", JDG 2000.) In the talk we will discuss some developments of this idea, in the case of 7 dimensions and G_{2} holonomy. In one direction we will consider boundary value problems, and reductions of the G_2 equation to 3 and 4 dimensions. In another direction we will consider adiabatic limits, making contact with the theory of maximal submanifolds in spaces of indefinite signature.

2:35–3:25 pm
Blaine Lawson (Stony Brook University)
Lagrangian Potential Theory and a Lagrangian Equation of Monge-Ampère Type
The point of this talk is to present a *Lagrangian potential theory,* which is in many ways analogous to classical pluripotential theory in complex analysis. I will also introduce a new Lagrangian differential operator of Monge-Ampère type. This ideas are new even in C^n. However, they apply quite generally– perhaps most importantly to symplectic manifolds equipped with a Gromov metric.
The Lagrangian Monge-Ampère operator is an explicit polynomial on Sym^2 *(TX)* whose principle branch defines the space of Lag- harmonics. Interestingly this operator depends only on the Laplacian and the

SKEW-Hermitian part of the Hessian if the function. The Dirichlet problem for this operator is solved in both the homogeneous and inhomogeneous cases. It is also solved for each of the other branches. We shall also look at the notions of Lagrangian plurisubharmonic and harmonic functions, Lagrangian convex manifolds and boundaries, and an analogue of the Levi problem. Parallels of this Lagrangian potential theory with standard (complex) pluripotential theory are emphasized.

3:45–4:35 pm
William Meeks (University of Massachusetts, Amherst)
Recent progress in the theory of CMC surfaces in 3-manifolds
In this talk I will report on some of the new results in the theory of constant mean curvature (CMC) surfaces M in Riemannian 3-manifolds N. I will mention just a few of the topics touched on in this talk. I first begin with the recent classification of CMC spheres in a homogeneous 3-manifold N and a sketch of its proof. The main result states that any two spheres in N with the same absolute mean curvature differ by an ambient isometry of N. Furthermore, the range of values of the mean curvature spheres are described in terms of the geometry of the universal cover X of N. In the case that X is diffeomorphic to R^3 then there exists a sphere of constant mean curvature H in N iff H is greater than half the Cheeger constant of X and otherwise there exists a sphere of constant mean curvature in N for every real number. These results generalize previous work of Hopf, of Abresch-Rosenberg and more recently of Danieil-Mira and of Meeks in the case of the Sol geometry.
Jointly with Tinaglia, we obtain curvature for embedded disks of fixed constant mean curvature H >0 in any fixed homogeneous 3-manifolds. In the R^3 setting this result implies that any complete embedded finite topology surface in R^3 of constant mean curvature is proper; this generalizes the previous work of Colding -Minicozzi in the case of minimal surfaces. Previous classification results then imply that the only complete embedded simply connected constant mean curvature surfaces in R^3 are the plane, the helicoid and round spheres. Another application of this work by Meeks-Tinaglia is to prove that complete embedded CMC surfaces of finite topology in a complete hyperbolic 3-manifold are proper if the mean curvature H is at least 1. On the other hand, Coskunuzer-Meeks-Tinaglia recently constructed for any H in [0,1) a non-proper, complete, stable embedded plane in hyperbolic 3-space having constant mean curvature H.
In 1982 Choi and Wang proved that an embedded closed minimal surface F in the the round three-sphere S^3 has a bound on its area that only depends on its genus; actually their result generalizes from the ambient space S^3 to any closed 3-manifold M with positive Ricci curvature. This result was then used by Choi and Schoen to prove the compactness of the moduli space of such examples of fixed genus g in M. Tinaglia and I have been able to give the following related result in the case of connected closed surfaces M embedded in any Riemannian homology 3- sphere manifold N:
Theorem: For any H>0 and non-negative integer g, there exists a constant A(N,g, H) such that any closed surface embedded in M of genus g and constant mean curvature H has area at most A(N,g,H). This area estimate lead to a natural compactification of the moduli space of all such embedded constant mean curvature H examples in N with genus at most g, when H lies in a fixed compact interval [a,b] of positive numbers, and under a compact deformation of the Riemannian metric on N.
The recent classification of properly embedded minimal surfaces of genus 0 in R^3 given by Meeks-Perez-Ros, Lopez-Ros, Colin and of Meeks-Rosenberg play a role in the above area estimates, as do the curvature estimates of Meeks-Tinaglia for certain complete embedded CMC surfaces in a Riemannian 3-manifold.
At the end of my talk I will present a brief survey of some recent results on the existence and classification of CMC foliations of closed and non-closed 3-manifolds.

Tuesday May 2, 2017

9:00–9:50 am
Camillo De Lellis (ETH Zurich)
The Onsager theorem
In the fifties John Nash astonished the geometers with his celebrated isometric embedding theorems. A folkloristic explanation of his first theorem is that you should be able to put any piece of paper in your pocket without crumpling or folding it, no matter how large it is. Ten years ago Laszlo, Szekelyhidi, and I discovered unexpected similarities with the behavior of some classical equations in fluid dynamics. Our remark sparked a series of discoveries and works which have gone in several directions. Among them the most notable is the recent proof of Phil Isett of a long-standing conjecture of Lars Onsager in the theory of turbulent flows.

10:05–10:55 am
Denis Auroux (University of California, Berkeley)
Speculations about homological mirror symmetry for affine hypersurfaces
The wrapped Fukaya category of an algebraic hypersurface H in $(C^*)^n$ is conjecturally related via homological mirror symmetry to the derived category of singularities of a toric Calabi-Yau manifold X, whose moment polytope is determined by the tropicalization of H. In this talk we will first explain the statement, and illustrate it for the case of the pair of pants; then we will outline some more speculative ideas about "relative" homological mirror symmetry for pairs $((C^*)^n, H)$ and wrapped Fukaya categories of higher-dimensional pairs of pants.

11:10–12:00 pm
John Pardon (Princeton University)
Liouville sectors and local open-closed map
I will describe joint work-in-progress with Sheel Ganatra and Vivek Shende. We investigate to what extent holomorphic curves and Lagrangian submanifolds can be "localized". One goal is to obtain a combinatorial presentation of the Fukaya category of any Stein manifold. Another corollary of our setup is a local-to-global argument for verifying Abouzaid's generation criterion for the Fukaya category

12:00–1:30 pm LUNCH

1:30–2:20 pm
William Minicozzi (Massachusetts Institute of Technology)
Level set method for motion by mean curvature
Modeling of a wide class of physical phenomena, such as crystal growth and flame propagation, leads to tracking fronts moving with curvature-dependent speed. When the speed is the curvature this leads to a degenerate elliptic nonlinear PDE. A priori solutions are only defined in a weak sense, but it turns out that they are always twice differentiable classical solutions. This result is optimal; their second derivative is continuous only in very rigid situations that have a simple geometric interpretation. The proof weaves together analysis and geometry. This is joint work with Toby Colding.

2:35–3:25 pm
Kenji Fukaya (Simons Center for Geometry and Physics, Stony Brook University)
Equivariant Floer homology
In this talk I will explain a construction of equivariant version of Lagrangian Floer homology with compact group action. I will explain some of the ideas to construct it and its (potential) applications.

3:45–4:35 pm
David Gabai (Princeton University)
The 4-Dimensional Light Bulb Theorem
We generalize the classical light bulb theorem to four dimensions. I.e. a smooth 2-sphere in S^2\times S^2 that is transverse to S^2\times 0 and homologous to 0\times S^2 is smoothly isotopically standard. We discuss generalizations to spheres in other spaces and applications.

4:50–5:40 pm
Artan Sheshmani (Harvard University Center of Mathematical Sciences)
Nested Hilbert schemes, local Donaldson-Thomas theory, Vafa-Witten and Seiberg-Witten invariants
We report on the recent rigorous and general construction of the deformation-obstruction theories and virtual fundamental classes of nested (flag) Hilbert scheme of one dimensional subschemes of a smooth projective algebraic surface. This construction will provide one with a general framework to compute a large class of already known invariants, such as Poincare invariants of Okonek et al, or the reduced local invariants of Kool and Thomas in the context of their local surface theory. We show how to compute the generating series of deformation invariants associated to the nested Hilbert schemes, and via exploiting the properties of vertex operators, prove that in some cases they are given by modular forms. We finally establish a connection between the Vafa-Witten invariants of local-surface threefolds (recently analyzed in full detail by Tanaka and Thomas) and such nested Hilbert schemes. This construction (via applying Mochizuki's wallcrossing techniques) enables one to obtain a relations between the generating series of Seiberg-Witten invariants of the surface, the Vafa-Witten invariants and some modular forms. This is joint work with Amin Gholampour and Shing-Tung Yau following arXiv:1701.08902 and arXiv:1701.08899.

5:55–6:45 pm
Tristan Collins (Harvard University)
The deformed Hermitian-Yang-Mills equation
Mirror symmetry predicts that the moduli space of complex structures/ special Lagrangians on one Calabi-Yau is dual to the moduli space of complexified forms/ stable bundles on the mirror Calabi-Yau. However, the precise definition of a complexified Kahler form/stable bundle has remained mysterious. I will discuss these notions in the setting of Strominger-Yau-Zaslow mirror symmetry, the connection to fully nonlinear PDEs and algebro-geometric stability. This is talk will discuss joint works with A. Jacob and S.-T. Yau, and G. Szekelyhidi.