

“We need you down here right away. We can stack the oranges, but we’re having trouble with the artichokes.”

—Ann Arbor Farmers Market to Thomas Hales

## The Trivial Notions Seminar Proudly Announces

### Lattice Packings of Spheres

A talk by  
Nathan Kaplan

#### Abstract

How densely can we pack non-overlapping spheres of fixed radius in  $n$ -dimensional Euclidean space? This classical problem has proven to be so difficult that we have a complete answer only in dimensions 1 (trivial), 2 (not easy; Thue 1890/Fejes Tóth 1940), and 3 (extremely challenging; Hales 1998).

If we restrict our packing so that the centers of our spheres lie on the points of a lattice, we can say much more. The optimal lattice packings are known in dimensions  $n \leq 8$ , and thanks to recent work of Cohn and Kumar building on results of Cohn and Elkies,  $n = 24$ .

In this talk we will sketch a proof that the hexagonal lattice gives the optimal packing in 2 dimensions and give an argument of Gauss that the face centered cubic is the optimal 3-dimensional lattice packing. We will describe results of Hermite, Minkowski, Voronoi and Mordell that have been useful in the study of lattice packings and also mention the amazing results showing that the Leech lattice gives the optimal lattice packing in 24 dimensions.

Thursday December 2<sup>nd</sup>, at 3:00 pm  
Science Center 507