# The Trivial Notions Seminar

Proudly Announces

# $K_2$ and $L$-functions of elliptic curves

A talk by

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**Abstract**

Euler in 1735 discovered that

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6},$$

and Dirichlet in 1839 proved that

$$1 - \frac{1}{3} - \frac{1}{5} + \frac{1}{7} + \frac{1}{9} - \frac{1}{11} - \frac{1}{13} + \frac{1}{15} + \cdots = \frac{1}{\sqrt{2}} \log(1 + \sqrt{2}).$$

We begin by re-interpreting these sums as special values of $L$-functions of number fields. The notion of $L$-functions has been vastly generalized and their special values are the subject of the celebrated conjectures of Birch-Swinnerton-Dyer, Beilinson and many others. We then focus on the next simplest (but incredibly rich) case: the $L$-function $L(E, s)$ of an elliptic curve $E$. In 1978, Bloch discovered a beautiful formula for $L(E, 2)$ in terms of Wigner's dilogarithm function (a generalization of the usual logarithm). We illustrate this formula with explicit examples and explain how Quillen's $K$-group $K_2(E)$ plays a surprising role in calculating $L(E, 2)$. No prior knowledge of algebraic $K$-theory or $L$-functions is required.

## Wednesday March $4^{\text{th}}$, at 1:00 pm

## Science Center 112